

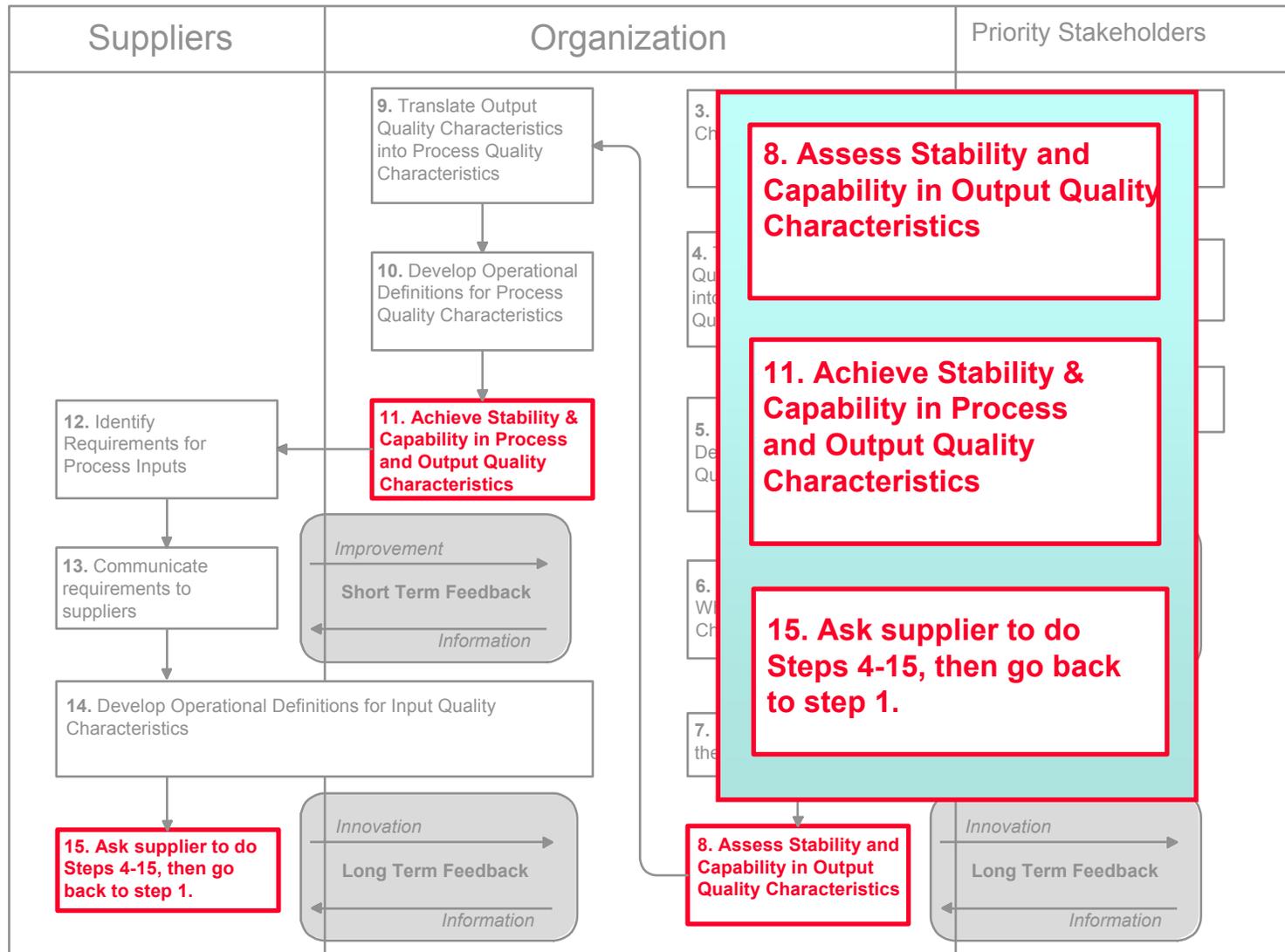
Module 4:

Assessing and Achieving Stability Using Averages Control Charts

Objectives

- ❑ **Construct averages control charts**
- ❑ **Assess process stability using a control chart**
- ❑ **Use rational subgrouping to make averages charts more useful**
- ❑ **Diagnose a process to differentiate between common and special causes**
- ❑ **To take action or not to take action? That is the question; or, what to do based on common or special causes.**

Where We Are in the Model



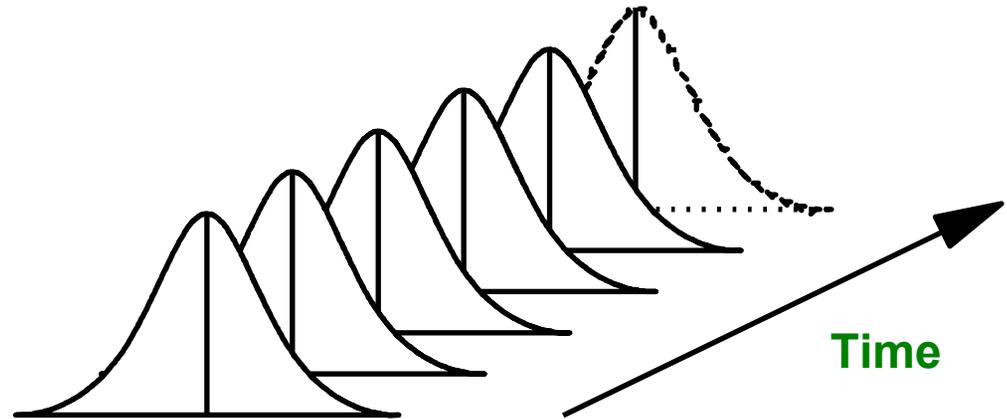
Relationship of Variation to Stability

Stable

In control

Predictable

Common causes only

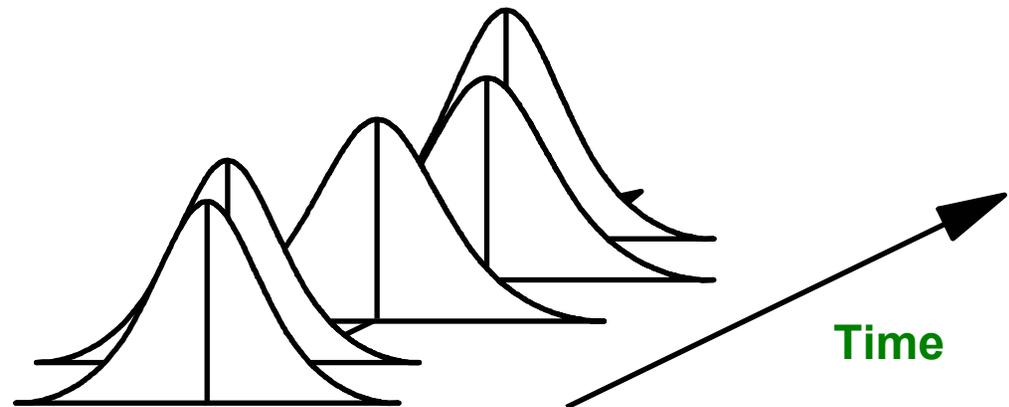


Unstable

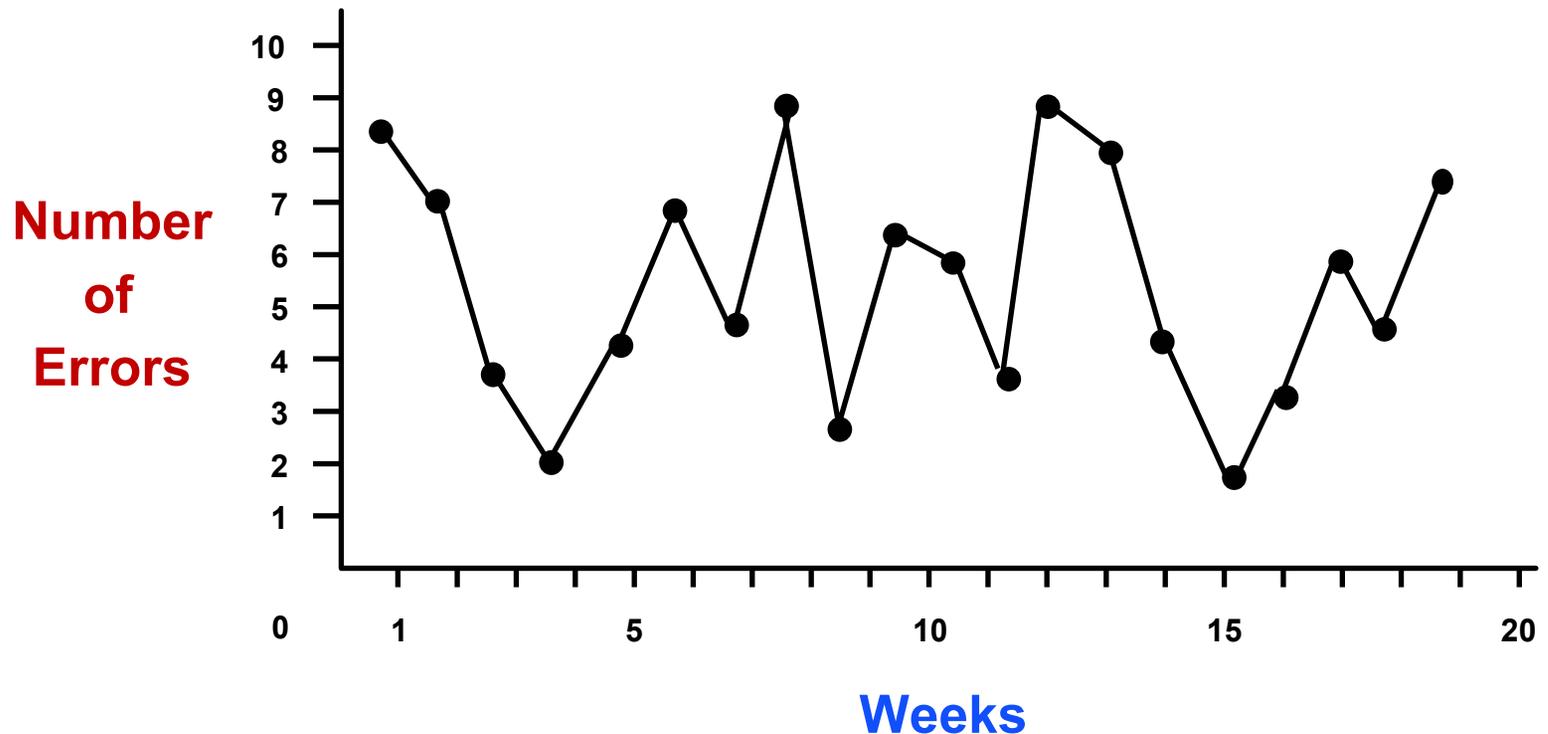
Out of control

Unpredictable

Special causes present



Definition of a Run Chart

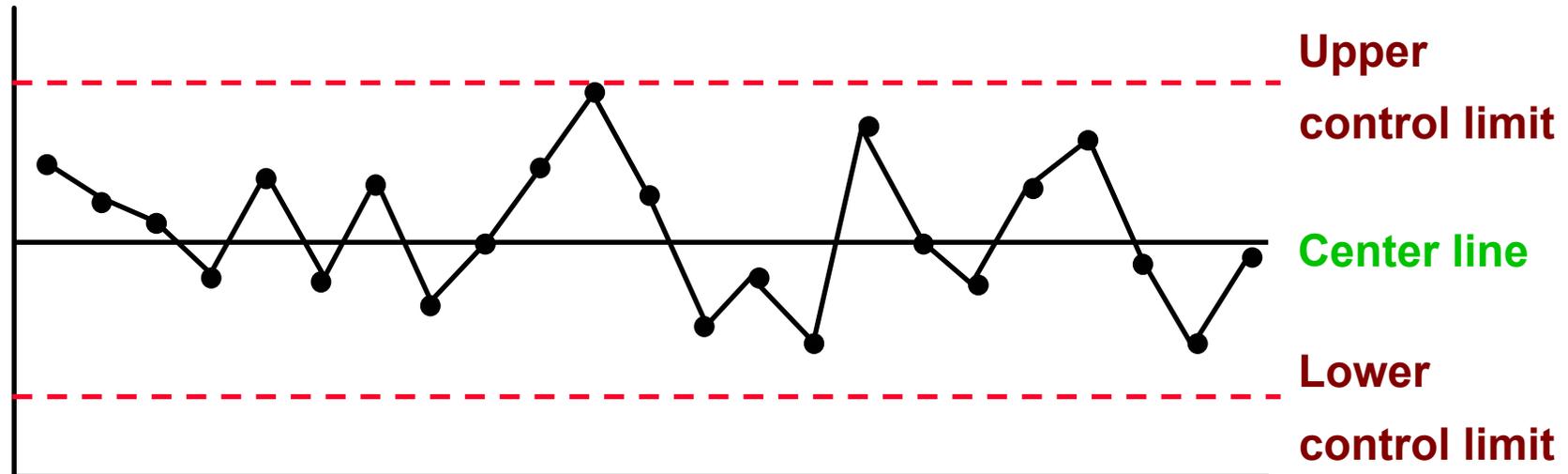


- A run chart is a line graph that shows data plotted over time

Uses of Run Charts

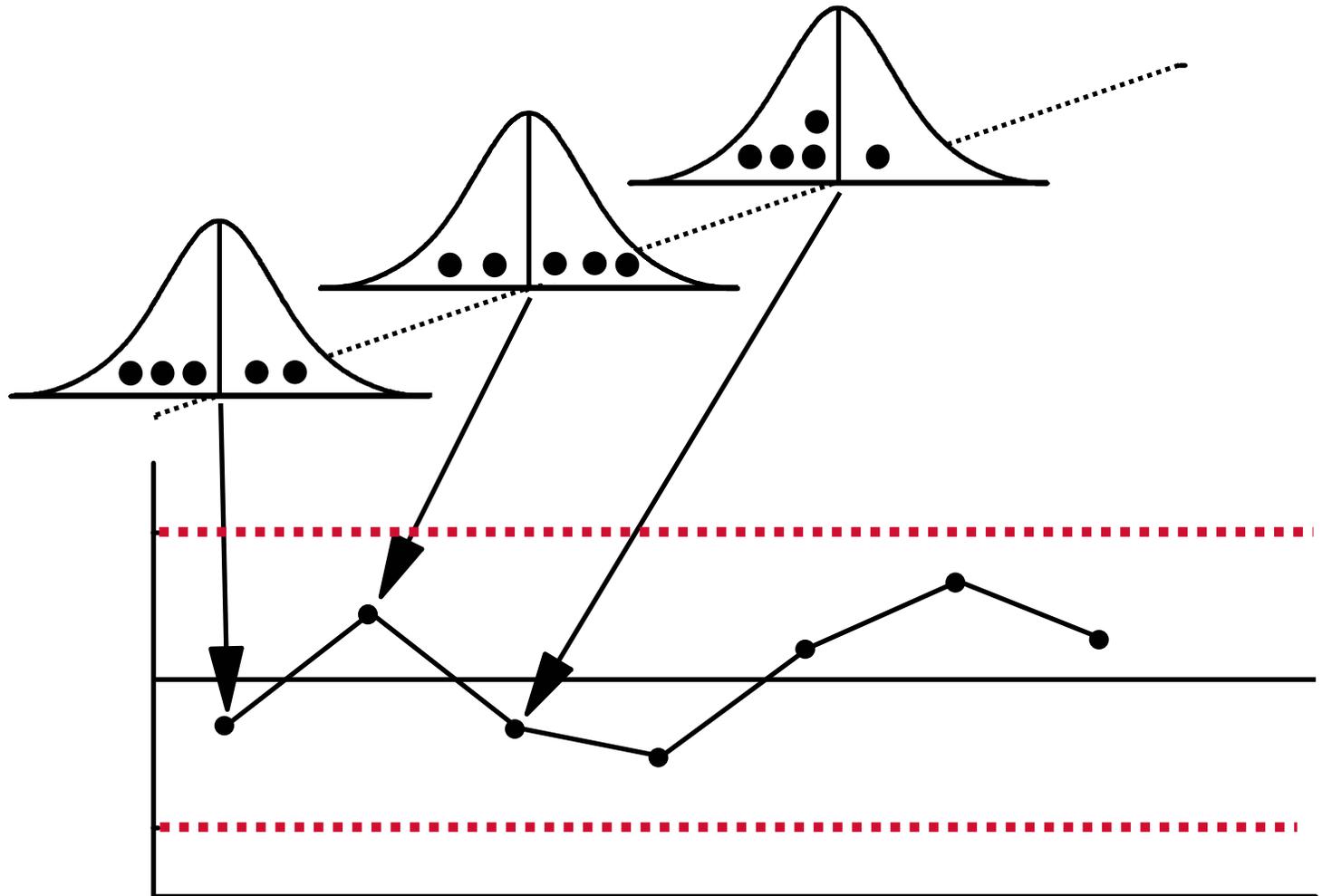
- ❑ **Study variation over time**
- ❑ **No basis for rational sampling**
- ❑ **Present data**

Definition and Elements of a Control Chart

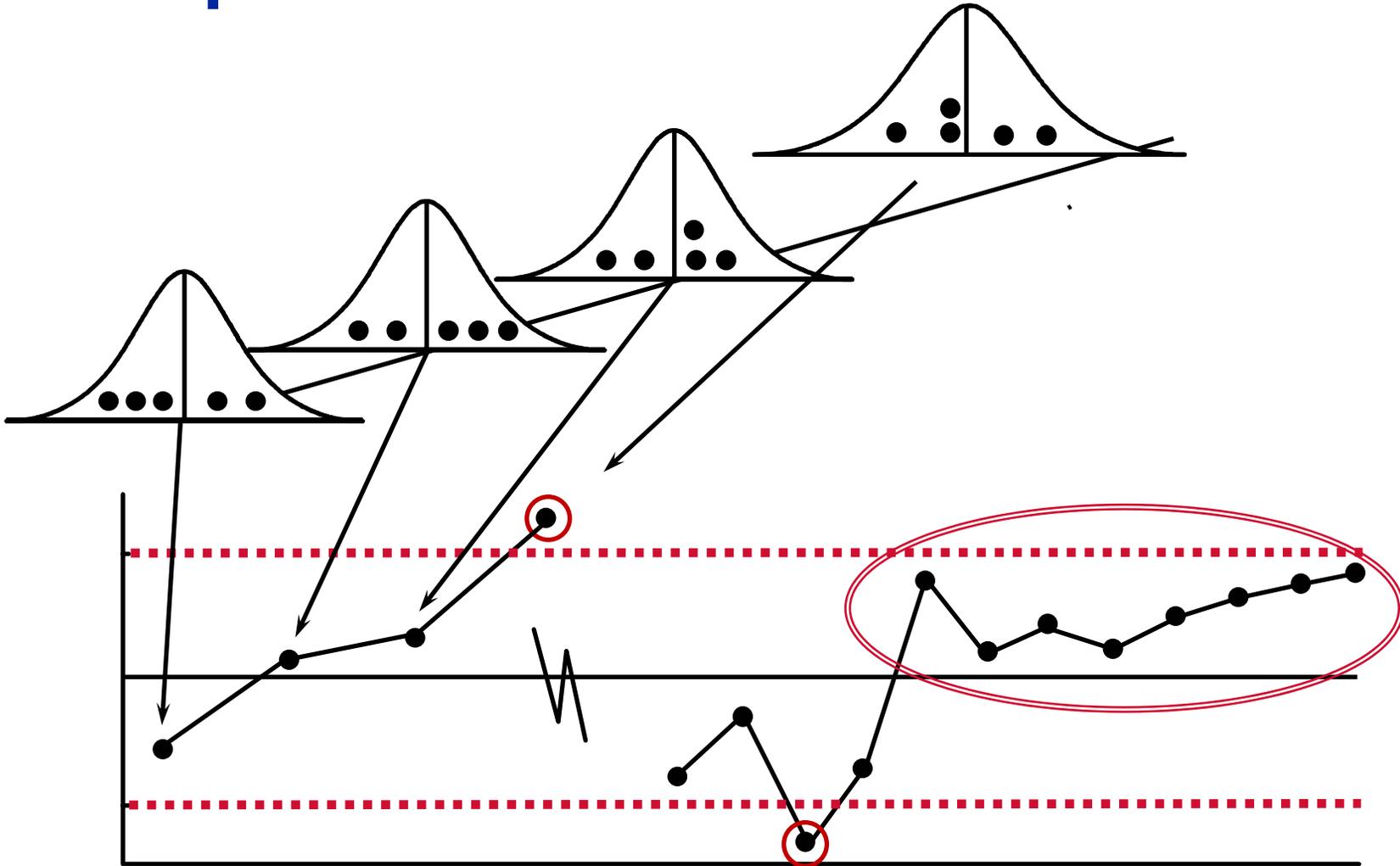


- **Control chart: graphic display of process variation over time used to distinguish between common and special causes**

Common Causes of Variation



Special Causes of Variation



Uses of Control Charts

- ❑ **Monitors process variation over time**
- ❑ **Differentiates between sources of variation**
- ❑ **Assesses results of changes**
- ❑ **Establishes the basis for determining process capability**

Control Charts

□ Averages Control Charts

\bar{X} -R

\bar{X} -s

\tilde{X} -R

□ Individuals Control Charts

XmR

p, np, c, u

Data for Averages Charts

- **Regular control chart data**
 - **Subgroup size independent of subgroup frequency**
- **Ability to select subgroup size:**
 - **Allows for the rational collection of several observations**
 - **Takes advantage of increased sensitivity of averages charts**

\bar{X} and R Control Charts

How to Construct \bar{X} and R Control Charts

1. Determine data collection procedures, subgroup period and size
2. Collect and record appropriate data
3. Calculate means (\bar{X}) and ranges (R) for each subgroup
4. Calculate grand mean ($\bar{\bar{X}}$) and average range (\bar{R})

How to Construct \bar{X} and R Control Charts

5. Calculate control limits for the \bar{X} and R control charts:

$$\begin{aligned} \text{X:} \quad \text{Upper Control Limit (UCL)} &= \bar{\bar{X}} + A_2 \bar{R} \\ \text{Lower Control Limit (LCL)} &= \bar{\bar{X}} - A_2 \bar{R} \end{aligned}$$

$$\begin{aligned} \text{R:} \quad \text{UCL} &= D_4 \bar{R} \\ \text{LCL} &= D_3 \bar{R} \end{aligned}$$

6. Select the scales

7. Plot the center line and the control limits for the \bar{X} and R charts

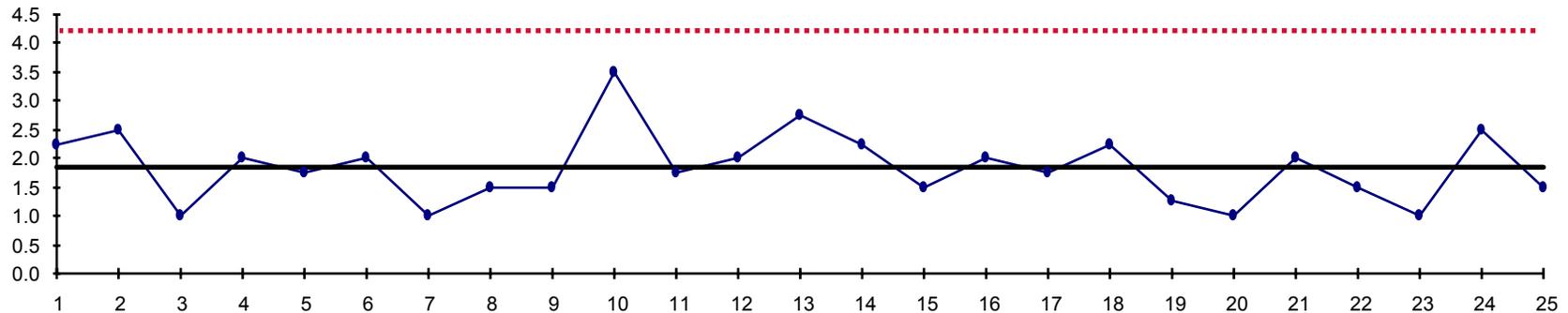
8. Plot the means and ranges on the control charts

9. Provide appropriate documentation

Provide Appropriate Documentation

- Who**
- How**
- What**
- Why**
- Where**

Distance from Limit of Contact Line



$\bar{X} = 42.013$
 $\bar{R} = 1.84$

$UCL_{\bar{x}} = 43.35$
 $LCL_{\bar{x}} = 40.67$

$UCL_R = 4.19$

Averages of four washers in 25 shots

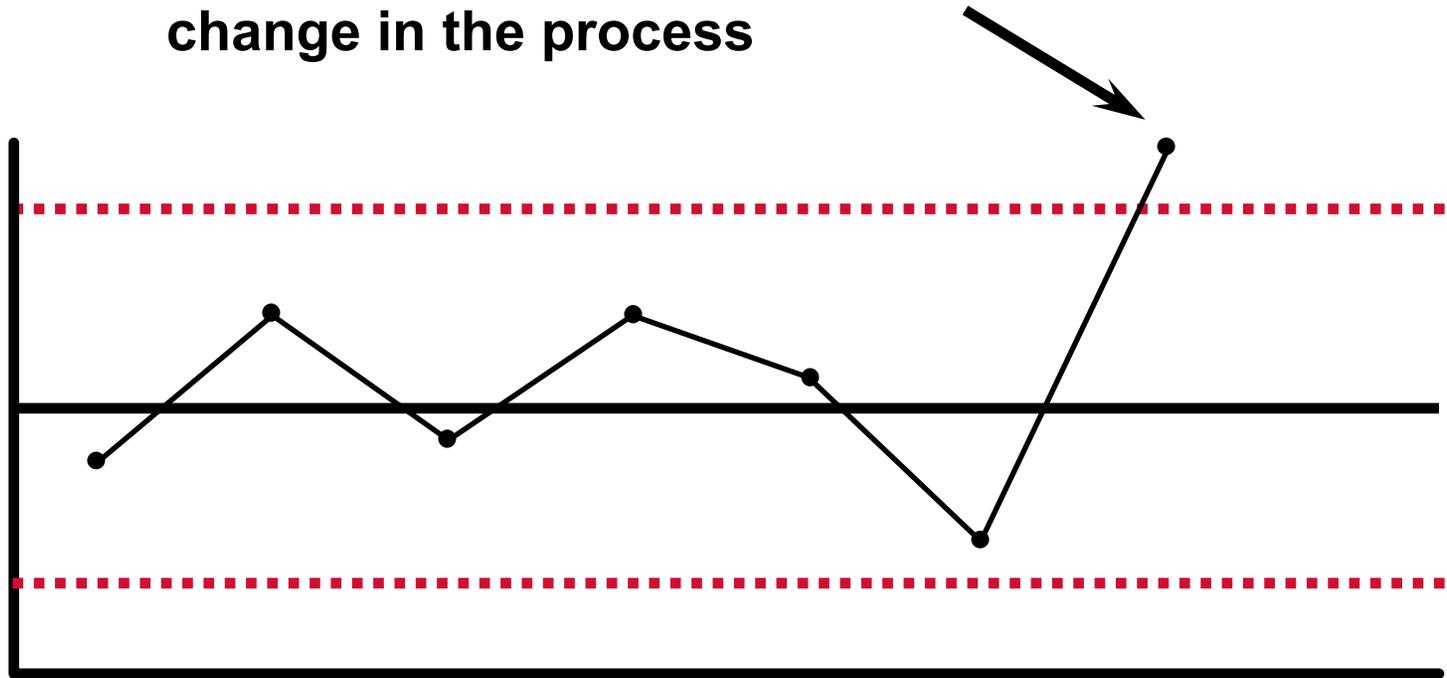
How To Interpret Signals on a Control Chart

Statistical guidelines used to indicate whether variation is due to common or special causes

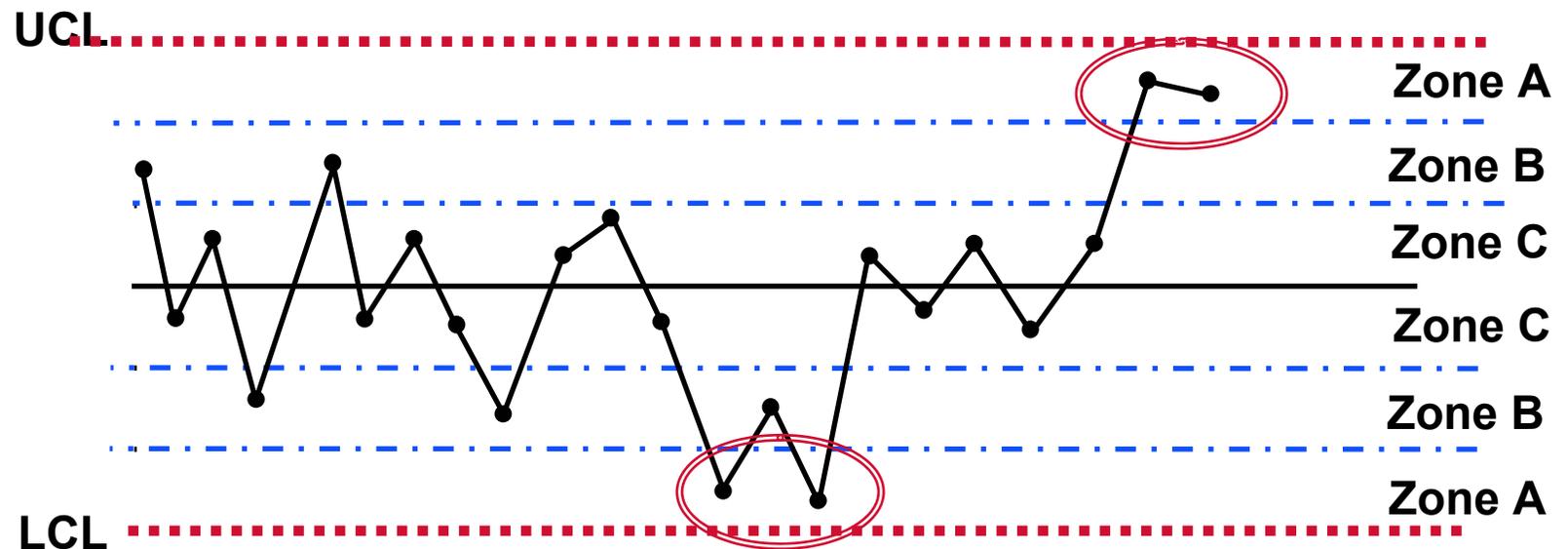
- **Common causes:** random pattern of variation, all points within the control limits
- **Special causes:** non-random pattern of variation, points beyond the control limits

Rule 1: A lack of control is indicated whenever a single point falls outside the control limits.

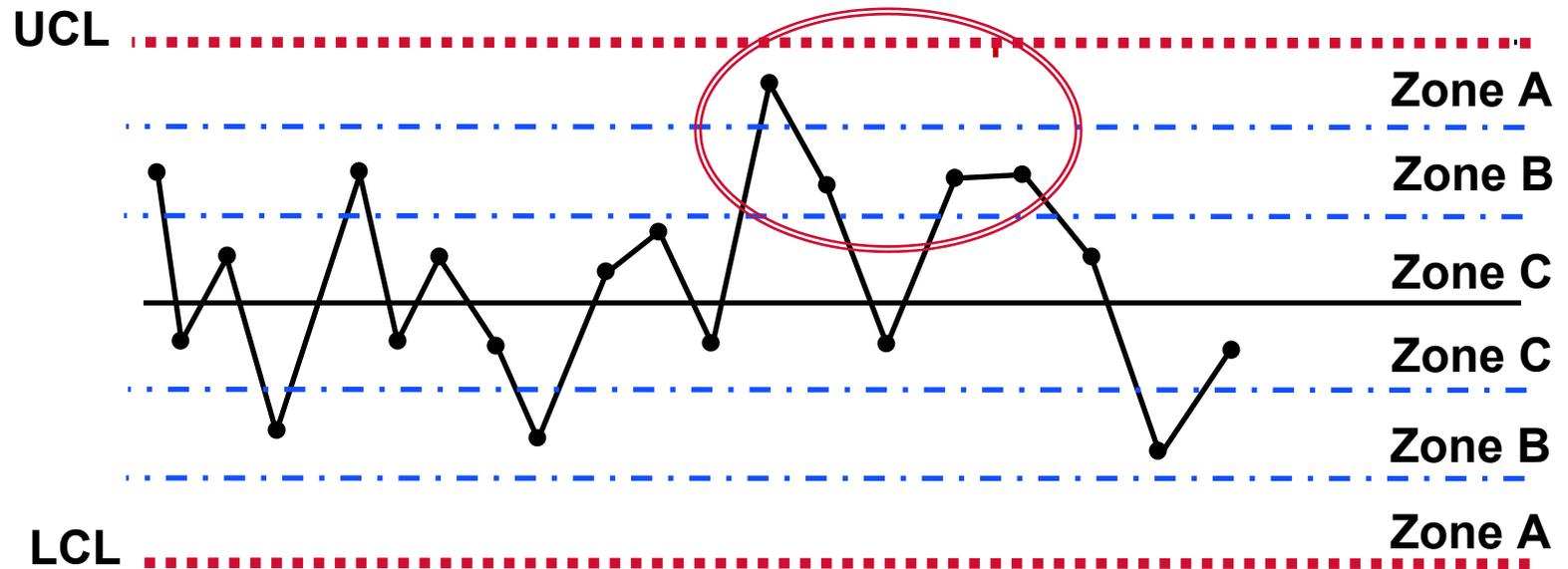
Unexpected points indicate a change in the process



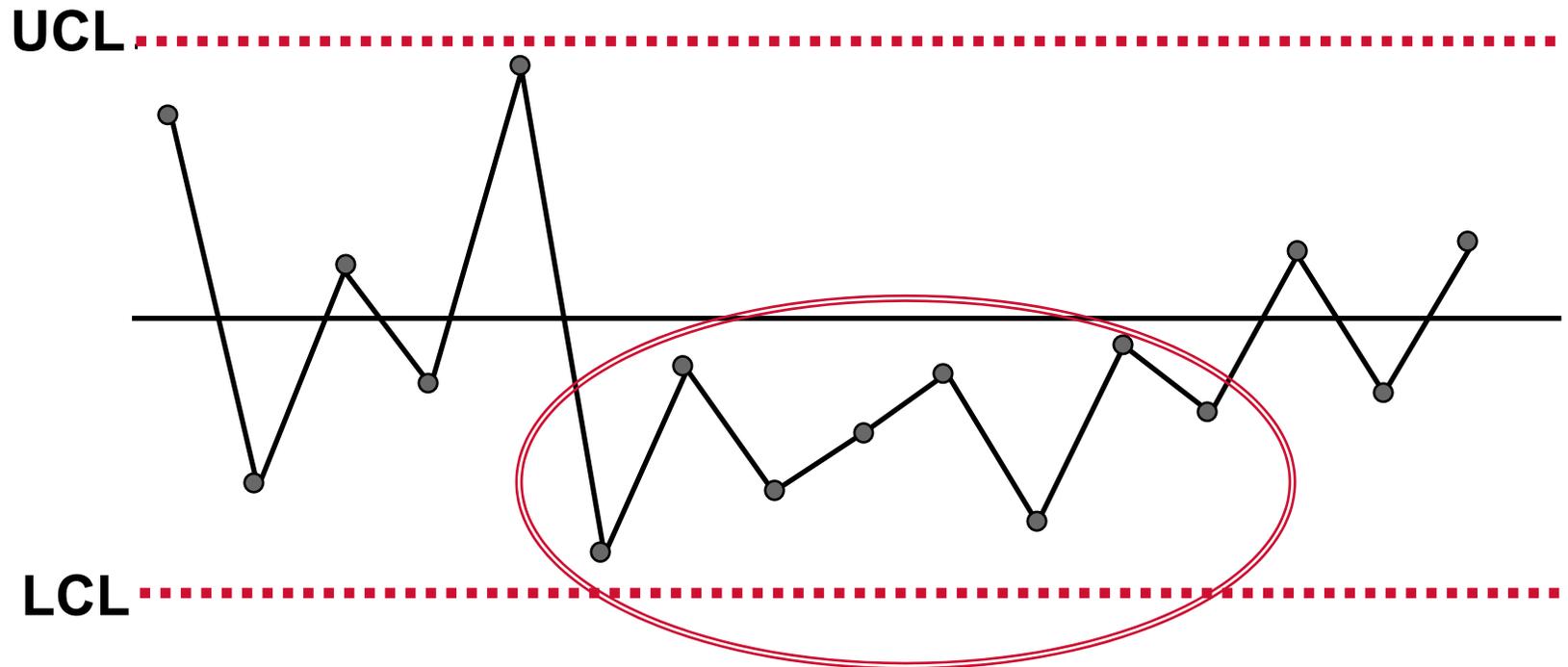
Rule 2: A lack of control is indicated whenever at least two out of three successive values fall on the same side of, and more than two sigma units away from the center line.



Rule 3: A lack of control is indicated whenever four out of five successive values fall on the same side of, and more than one sigma unit away from, the center line



Rule 4: A lack of control is indicated whenever at least eight successive values fall on the same side of the center line



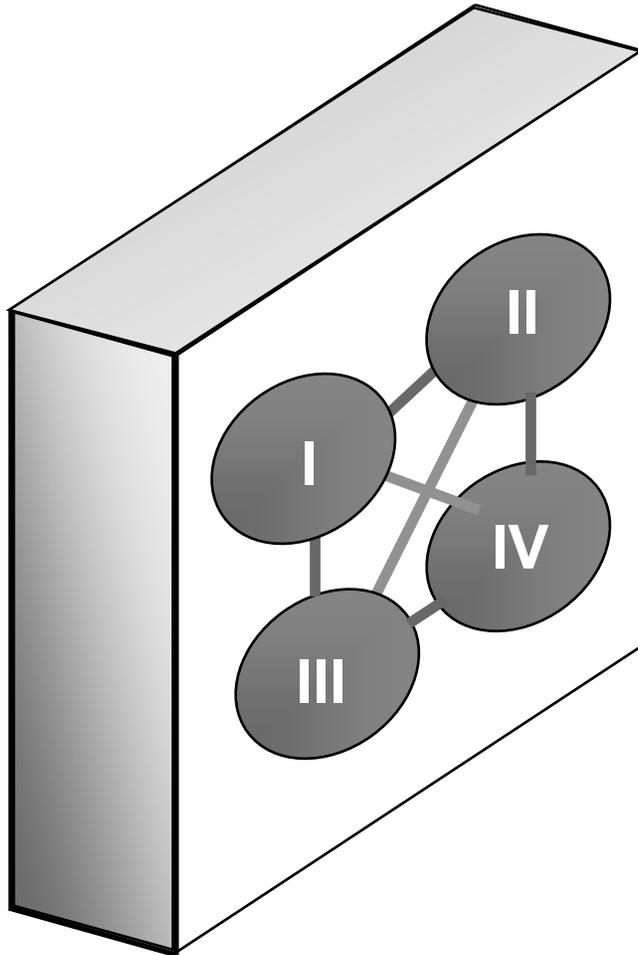
Comparison to Other Rules

- ❑ **Trend - six or eight points going up or down**
- ❑ **Cycles - repeating patterns**
- ❑ **Hugging the center line - 90% of points in middle 1/3 of the distribution**
- ❑ **Disproportionate number of points in outer zones**

Rational Subgrouping

- **Ball Socket Data Example**

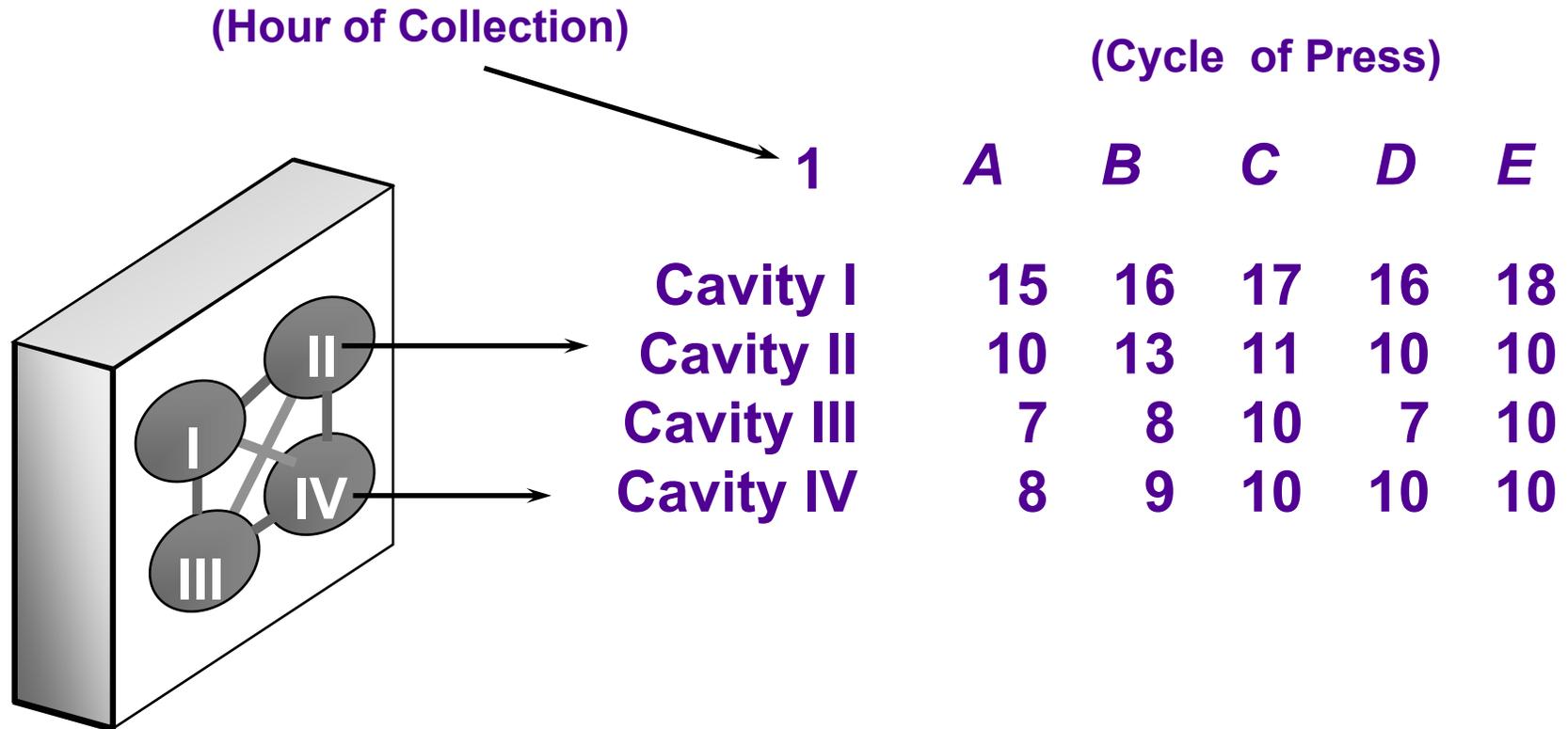
Socket Mold



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- Injection molding process produces sockets
- Each cycle of the press produces four sockets from the mold
- Four samples taken per day
- Each sample consists of parts produced by five consecutive cycles of the press
- Numbers represent thickness of the socket

Sample Data Recording Detail



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First Organization of Ball Socket Data

Hour 1						Hour 2							
		Cycle							Cycle				
		A	B	C	D	E		A	B	C	D	E	
CAV	I	15	16	17	16	18	CAV	I	13	18	15	15	15
	II	10	13	11	10	10		II	9	10	11	8	9
	III	7	8	10	7	10		III	7	11	10	10	9
	IV	8	9	10	10	10		IV	10	13	13	10	9
X-bar	10	11.5	12	10.75	12	X-bar	9.75	13	12.25	10.75	10.5		
R	8	8	7	9	8	R	6	8	5	7	6		
Hour 3						Hour 4							
		Cycle							Cycle				
		A	B	C	D	E		A	B	C	D	E	
CAV	4						CAV	5					
	I	14	15	15	15	14		I	12	13	13	12	13
	II	10	10	11	11	10		II	8	7	8	7	7
	III	7	9	12	10	10		III	5	6	8	5	4
IV	11	12	11	10	13	IV	4	4	5	3	4		
X-bar	10.5	11.5	12.25	11.5	11.75	X-bar	7.25	7.5	8.5	6.75	7		
R	7	6	4	5	4	R	8	9	8	9	9		

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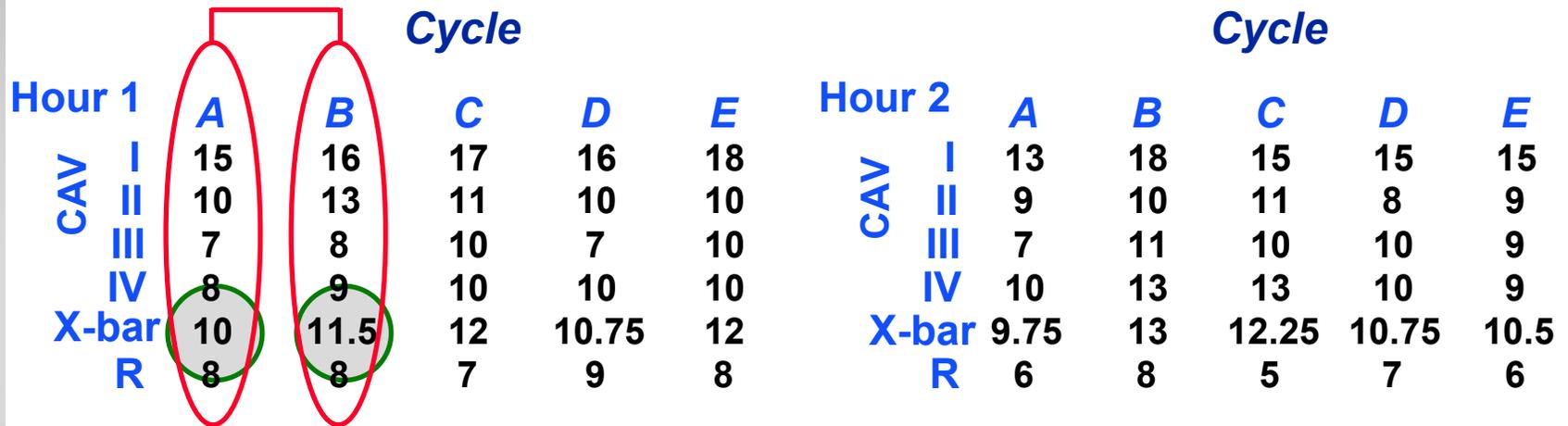
First Organization of Ball Socket Data

		Cycle							Cycle				
Hour 1		A	B	C	D	E	Hour 2		A	B	C	D	E
CAV	I	15	16	17	16	18	CAV	I	13	18	15	15	15
	II	10	13	11	10	10		II	9	10	11	8	9
	III	7	8	10	7	10		III	7	11	10	10	9
	IV	8	9	10	10	10		IV	10	13	13	10	9
X-bar		10	11.5	12	10.75	12	X-bar		9.75	13	12.25	10.75	10.5
R		8	8	7	9	8	R		6	8	5	7	6

- How do these subgroups differ in context?
 - 🕒 Hour-to-Hour
 - 🕒 Cycle-to-Cycle
 - 🕒 Cavity-to-Cavity

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First Organization of Ball Socket Data



- How do these subgroups differ in context?
 - 🕒 Hour-to-Hour
 - 🕒 Cycle-to-Cycle
 - 🕒 Cavity-to-Cavity

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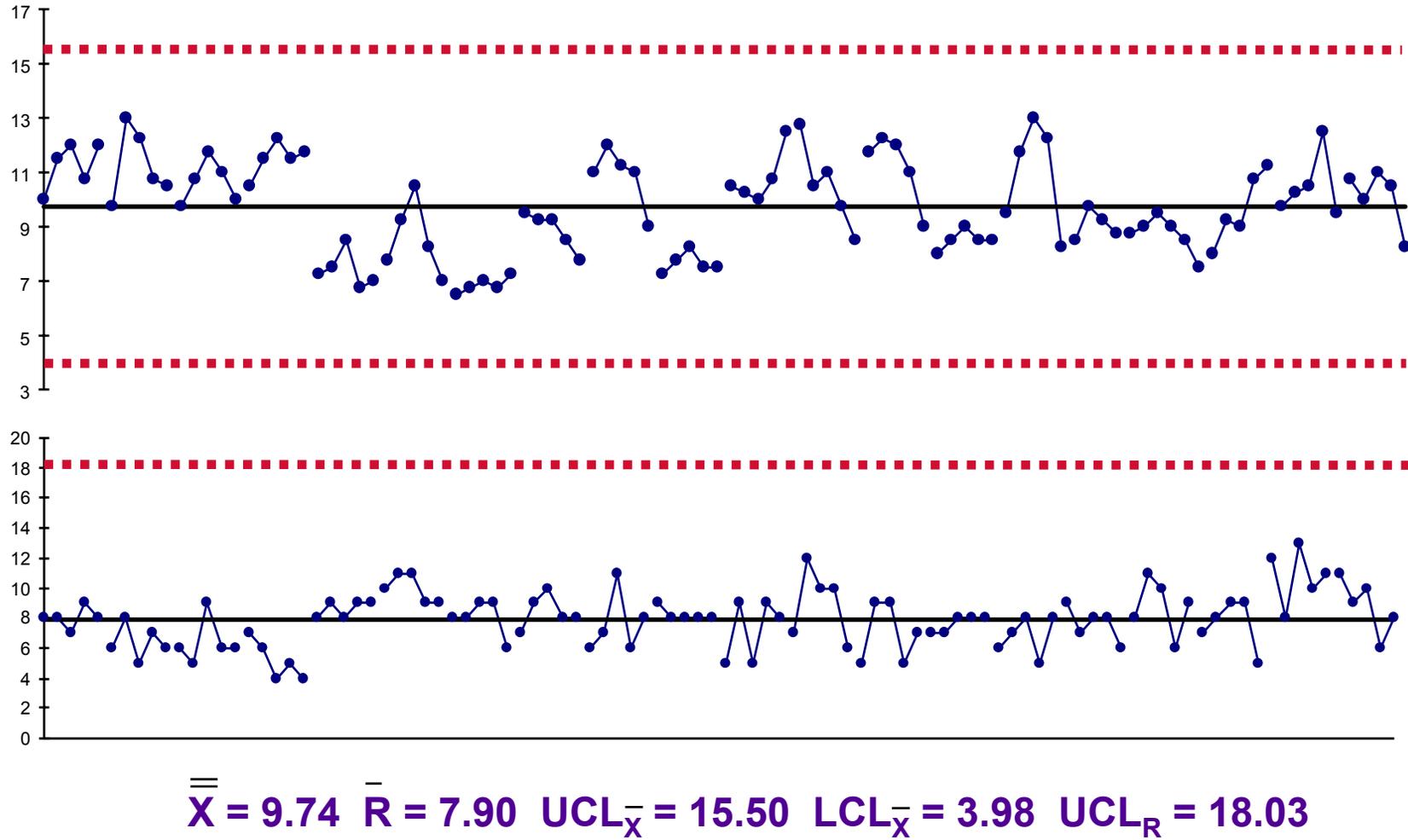
First Organization of Ball Socket Data

Hour 4						Hour 5						
	Cycle						Cycle					
	A	B	C	D	E		A	B	C	D	E	
CAV	I	14	15	15	15	14	I	12	13	13	12	13
	II	10	10	11	11	10	II	8	7	8	7	7
	III	7	9	12	10	10	III	5	6	8	5	4
	IV	11	12	11	10	13	IV	4	4	5	3	4
X-bar	10.5	11.5	12.25	11.5	11.75	X-bar	7.25	7.5	8.5	6.75	7	
R	7	6	4	5	4	R	8	9	8	9	9	

- What differences exist within the subgroups?
 - Hour-to-Hour
 - Cycle-to-Cycle
 - Cavity-to-Cavity

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Control Chart - First Organization



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First Organization Produced Charts

Which Ask:

- ❑ **Are there detectable differences from Hour-to Hour?**
- ❑ **Are there detectable differences from Cycle-to-Cycle?**
- ❑ **Are the Cavity-to-Cavity differences consistent?**

Second Organization of Ball Socket Data

Hour 1								Hour 2									
Cycle								Cycle									
	A	B	C	D	E	X-bar	R		A	B	C	D	E	X-bar	R		
CAV	I	15	16	17	16	18	16.4	3	CAV	I	13	18	15	15	15	15.2	5
	II	10	13	11	10	10	10.8	3		II	9	10	11	8	9	9.4	3
	III	7	8	10	7	10	8.4	3		III	7	11	10	10	9	9.4	4
	IV	8	9	10	10	10	9.4	2		IV	10	13	13	10	9	11	4
4	A	B	C	D	E	X-bar	R	5	A	B	C	D	E	X-bar	R		
I	14	15	15	15	14	14.6	1	I	12	13	13	12	13	12.6	1		
II	10	10	11	11	10	10.4	1	II	8	7	8	7	7	7.4	1		
III	7	9	12	10	10	9.6	5	III	5	6	8	5	4	5.6	4		
IV	11	12	11	10	13	11.4	3	IV	4	4	5	3	4	4	2		

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Second Organization of Ball Socket Data

Hour 1								Hour 2							
Cycle								Cycle							
CAV	A	B	C	D	E	X-bar	R	CAV	A	B	C	D	E	X-bar	R
I	15	16	17	16	18	16.4	3	I	13	18	15	15	15	15.2	5
II	10	13	11	10	10	10.8	3	II	9	10	11	8	9	9.4	3
III	7	8	10	7	10	8.4	3	III	7	11	10	10	9	9.4	4
IV	8	9	10	10	10	9.4	2	IV	10	13	13	10	9	11	4

- What changes between these subgroups?
 - Hour
 - Cycle
 - Cavity

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Second Organization of Ball Socket Data

Hour 1								Hour 2									
		Cycle								Cycle							
		A	B	C	D	E	X-bar	R			A	B	C	D	E	X-bar	R
CAV	I	15	16	17	16	18	16.4	3	CAV	I	13	18	15	15	15	15.2	5
	II	10	13	11	10	10	10.8	3		II	9	10	11	8	9	9.4	3
	III	7	8	10	7	10	8.4	3		III	7	11	10	10	9	9.4	4
	IV	8	9	10	10	10	9.4	2		IV	10	13	13	10	9	11	4

- What differences exist within the subgroups?
 - Hour-to-Hour
 - Cycle-to-Cycle
 - Cavity-to-Cavity

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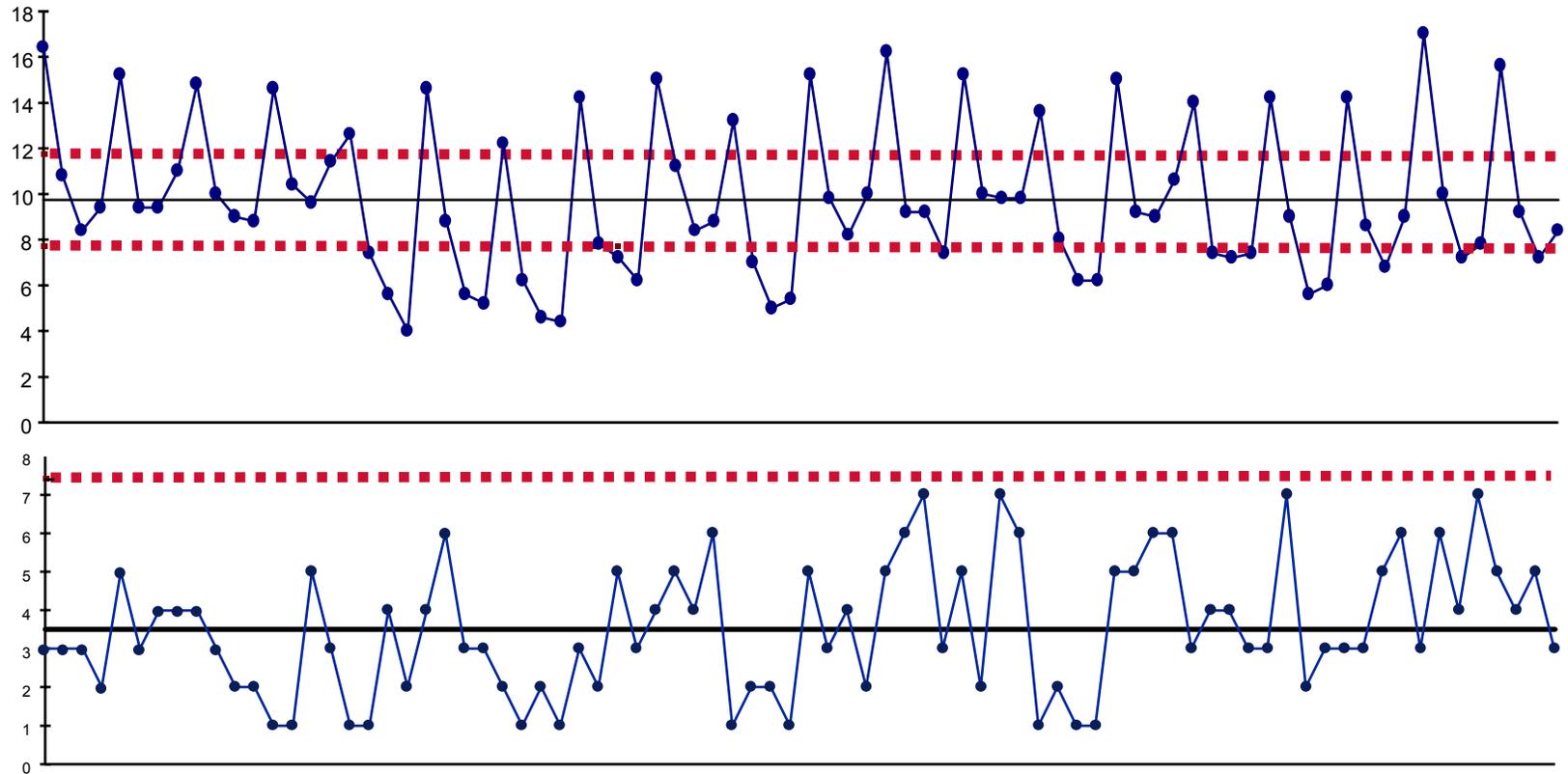
Second Organization of Ball Socket Data

Hour 1								Hour 2							
Cycle								Cycle							
	A	B	C	D	E	X-bar	R		A	B	C	D	E	X-bar	R
CAV I	15	16	17	16	18	16.4	3	CAV I	13	18	15	15	15	15.2	5
CAV II	10	13	11	10	10	10.8	3	CAV II	9	10	11	8	9	9.4	3
CAV III	7	8	10	7	10	8.4	3	CAV III	7	11	10	10	9	9.4	4
CAV IV	8	9	10	10	10	9.4	2	CAV IV	10	13	13	10	9	11	4

- What is different between these subgroups?
 - Hour
 - Cycle
 - Cavity

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Control Chart - Second Organization



$\bar{\bar{X}} = 9.74$ $\bar{R} = 3.51$ $UCL_{\bar{X}} = 11.77$ $LCL_{\bar{X}} = 7.71$ $UCL_R = 7.42$

(First: $\bar{\bar{X}} = 9.74$ $\bar{R} = 7.90$ $UCL_{\bar{X}} = 15.50$ $LCL_{\bar{X}} = 3.98$ $UCL_R = 18.03$)

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Second Organization Produced Charts Which Ask:

- **(1) Are there detectable differences from Hour-to-Hour?**
- **(2) Are the Cycle-to-Cycle differences consistent?**
- **(3) Are there detectable differences from Cavity-to-Cavity?**

Third Organization of the Ball Socket Data

Cavity I

	A	B	C	D	E	X-bar	R
1	15	16	17	16	18	16.4	3
4	14	15	15	15	14	14.6	1
7	12	12	13	13	11	12.2	2
10	13	13	14	13	13	13.2	1
13	13	18	18	14	13	15.2	5
16	15	14	15	14	12	14	3
19	17	15	18	18	17	17	3

	A	B	C	D	E	X-bar	R
2	13	18	15	15	15	15.2	5
5	12	13	13	12	13	12.6	1
8	13	15	16	14	13	14.2	3
11	13	16	13	16	18	15.2	5
14	13	13	14	14	14	13.6	1
17	14	16	14	13	14	14.2	3
20	18	16	17	14	13	15.6	5

	A	B	C	D	E	X-bar	R
3	14	14	18	14	14	14.8	4
6	14	15	17	14	13	14.6	4
9	15	16	17	14	13	15	4
12	16	18	18	16	13	16.2	5
15	13	15	18	15	14	15	5
18	12	15	15	15	14	14.2	3

Cavity II

	A	B	C	D	E	X-bar	R
1	10	13	11	10	10	10.8	3
4	10	10	11	11	10	10.4	1
7	6	6	6	6	7	6.2	1
10	7	8	7	6	7	7	2
13	11	10	9	10	10	10	2
16	6	7	8	6	10	7.4	4
19	10	10	12	12	6	10	6

	A	B	C	D	E	X-bar	R
2	9	10	11	8	9	9.4	3
5	8	7	8	7	7	7.4	1
8	9	8	7	7	8	7.8	2
11	10	8	10	10	11	9.8	3
14	7	8	9	8	8	8	2
17	9	10	13	7	6	9	7
20	8	10	11	10	7	9.2	4

	A	B	C	D	E	X-bar	R
3	9	10	10	12	9	10	3
6	7	11	12	8	6	8.8	6
9	11	13	11	13	8	11.2	5
12	13	8	8	10	7	9.2	6
15	7	12	10	10	7	9.2	5
18	9	7	8	10	9	8.6	3

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Third Organization of the Ball Socket Data

Cavity 1

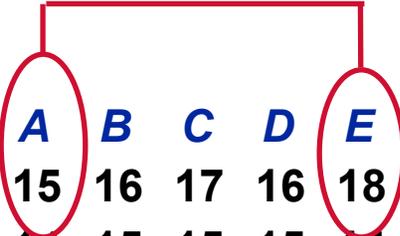
	A	B	C	D	E	X-bar	R		A	B	C	D	E	X-bar	R
1	15	16	17	16	18	16.4	3	2	13	18	15	15	15	15.2	5
4	14	15	15	15	14	14.6	1	5	12	13	13	12	13	12.6	1
7	12	12	13	13	11	12.2	2	8	13	15	16	14	13	14.2	3
10	13	13	14	13	13	13.2	1	11	13	16	13	16	18	15.2	5
13	13	18	18	14	13	15.2	5	14	13	13	14	14	14	13.6	1
16	15	14	15	14	12	14	3	17	14	16	14	13	14	14.2	3
19	17	15	18	18	17	17	3	20	18	16	17	14	13	15.6	5

□ How do these subgroups differ in context?

- ① Hour
- ① Cycle
- ① Cavity

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Third Organization of the Socket Data



	A	B	C	D	E	X-bar	R		A	B	C	D	E	X-bar	R
1	15	16	17	16	18	16.4	3	2	13	18	15	15	15	15.2	5
4	14	15	15	15	14	14.6	1	5	12	13	13	12	13	12.6	1
7	12	12	13	13	11	12.2	2	8	13	15	16	14	13	14.2	3
10	13	13	14	13	13	13.2	1	11	13	16	13	16	18	15.2	5
13	13	18	18	14	13	15.2	5	14	13	13	14	14	14	13.6	1
16	15	14	15	14	12	14	3	17	14	16	14	13	14	14.2	3
19	17	15	18	18	17	17	3	20	18	16	17	14	13	15.6	5

- **What differences exist within this subgroup?**
 - ① **Hour-to-Hour**
 - ① **Cycle-to-Cycle**
 - ① **Cavity-to-Cavity**

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Third Organization of the Socket Data

Cavity I

	A	B	C	D	E	X-bar	R
1	15	16	17	16	18	16.4	3
4	14	15	15	15	14	14.6	1
7	12	12	13	13	11	12.2	2
10	13	13	14	13	13	13.2	1
13	13	18	18	14	13	15.2	5
16	15	14	15	14	12	14	3
19	17	15	18	18	17	17	3

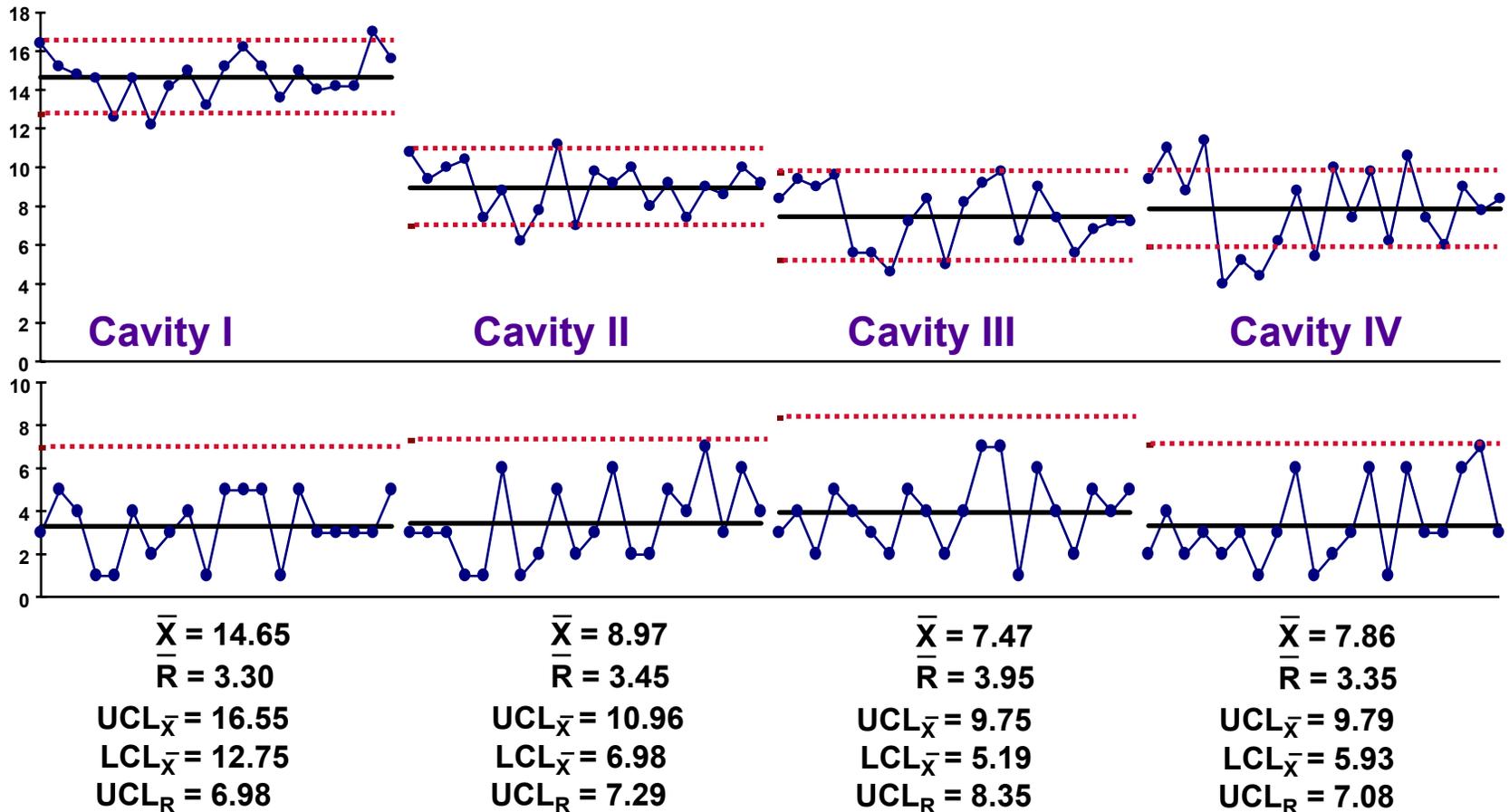
Cavity II

	A	B	C	D	E	X-bar	R
1	10	13	11	10	10	10.8	3
4	10	10	11	11	10	10.4	1
7	6	6	6	6	7	6.2	1
10	7	8	7	6	7	7	2
13	11	10	9	10	10	10	2
16	6	7	8	6	10	7.4	4
19	10	10	12	12	6	10	6

- What differences exist between these subgroups?
 - 🕒 Hour
 - 🕒 Cycle
 - 🕒 Cavity

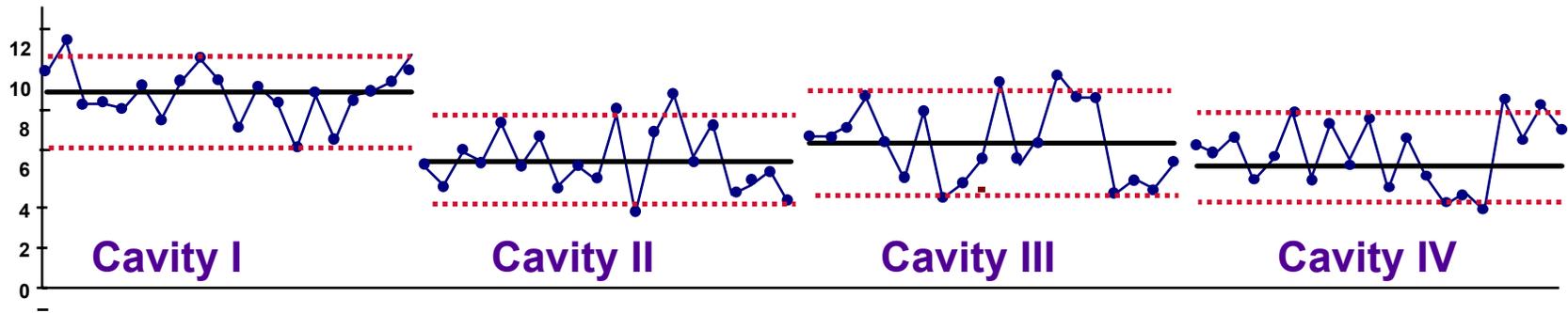
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Control Chart - Third Organization



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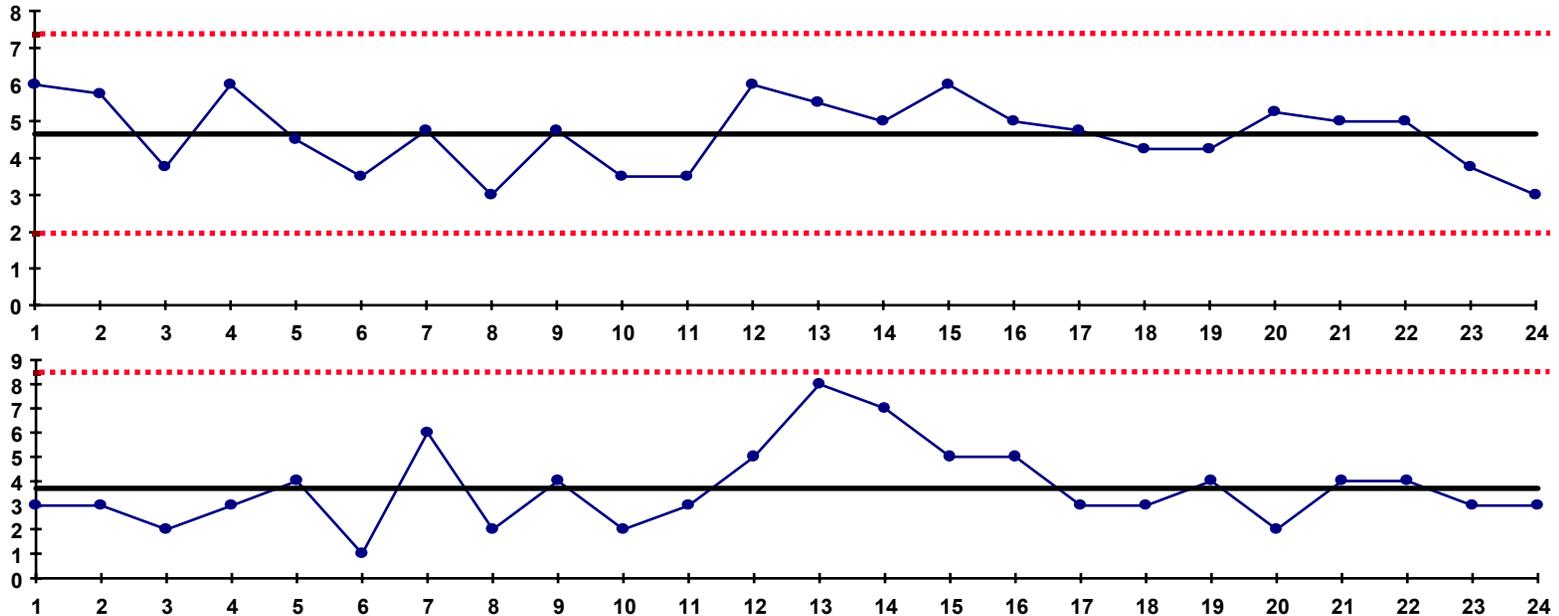
Socket Data Following Mold Adjustment



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Operator's Control Chart

	January 3									January 4									1/5	1/6	1/7			
Cavity I	4	4	3	7	4	3	3	3	3	3	2	8	5	3	9	3	4	3	3	4	4	4	3	3
Cavity II	7	6	4	6	7	4	9	4	6	5	5	6	10	9	6	8	7	6	7	6	7	7	6	5
Cavity III	6	6	3	4	4	4	3	2	3	3	2	7	5	6	5	5	4	5	4	5	6	3	3	2
Cavity IV	7	7	5	7	3	3	4	3	7	3	5	3	2	2	4	4	4	3	3	6	3	6	3	2
Averages	6	5.8	3.8	6	4.5	3.5	4.8	3	4.8	3.5	3.5	6	5.5	5	6	5	4.8	4.2	4.2	5.2	5	5	3.8	3
Ranges	3	3	2	3	4	1	6	2	4	2	3	5	8	7	5	5	3	3	4	2	4	3	3	



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Data Collection: Rational Subgrouping

Subgrouping Principles:

- ❑ Never knowingly subgroup unlike things
- ❑ Minimize variation within each subgroup
- ❑ Maximize variation between subgroups
- ❑ Average across noise, not across signals
- ❑ Treat the chart in accordance with the use of the data
- ❑ Establish operational definitions for the sampling procedure

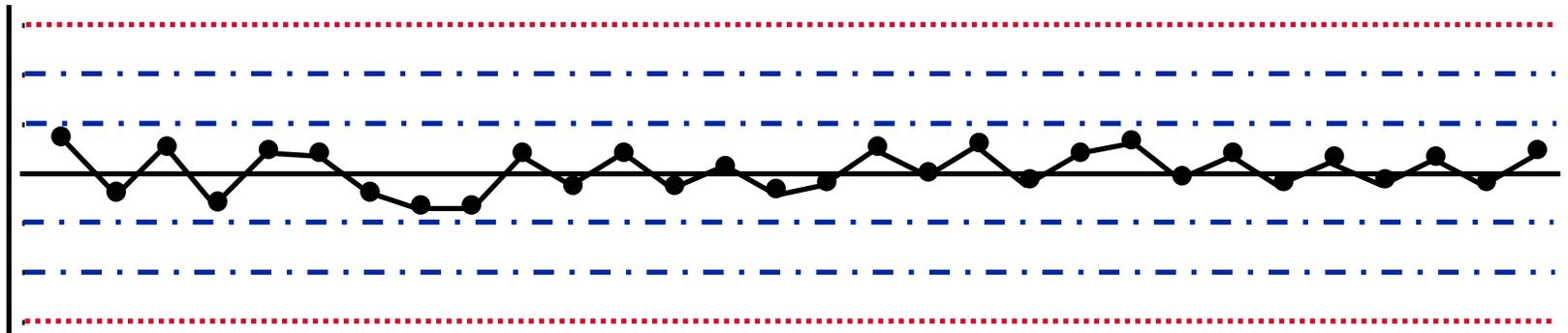
(Wheeler and Chambers, 1992)

Subgroup Size and Frequency - What is Appropriate

- **Subgroup Size**
 - **Small enough to minimize opportunities for special causes**
 - **Large enough to reflect the process distribution**
 - **Appropriate to detect process changes**
 - **Small enough to keep costs reasonable**
- **Subgroup Frequency**
 - **Based on process knowledge**
 - **Costs/Benefits**

A Potential Problem With Nonrational Samples

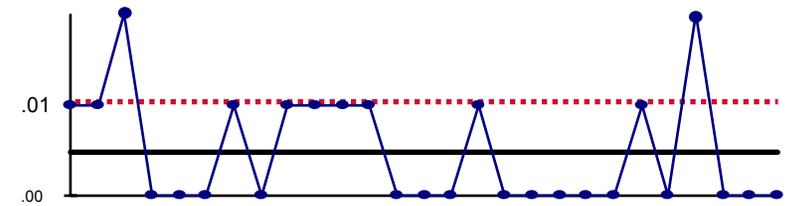
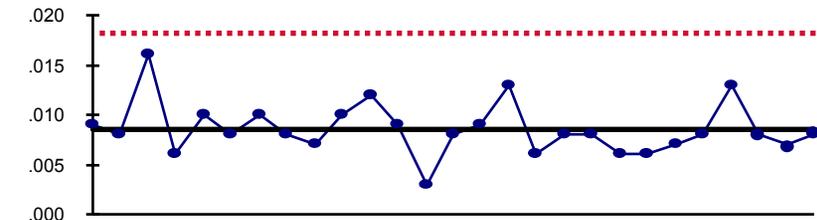
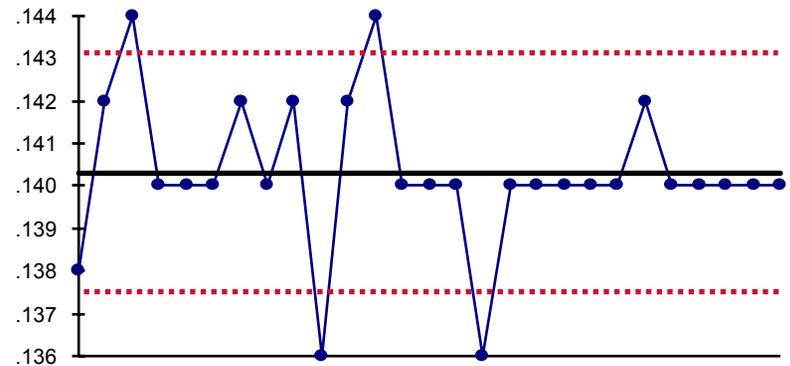
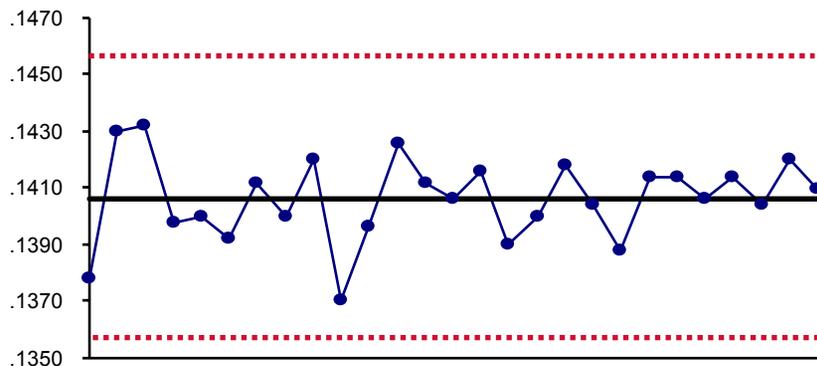
Lack of Stratification



Inadequate Measurement Units

1	.140.143.137.134.135	.1378	.009
2	.138.143.143.145.146	.1430	.008
3	.139.133.147.148.149	.1432	.016
4	.143.141.137.138.140	.1398	.006
5	.142.142.145.135.136	.1400	.010

1	.14	.14	.14	.13	.14	.138	.01
2	.14	.14	.14	.14	.15	.142	.01
3	.14	.13	.15	.15	.15	.144	.02
4	.14	.14	.14	.14	.14	.140	.00
5	.14	.14	.14	.14	.14	.140	.00



When to Change Control Limits

- ❑ **Process changes**
- ❑ **Replacing trial limits**
- ❑ **Removing out-of-control points**

When Improvements Can Be Realized

- **For Special Causes**
 - **Investigate immediately to identify source**
 - **Determine specific steps to remove or incorporate specific causes**
- **For Common Causes**
 - **Investigate process**
 - **Determine system changes needed**
- **Avoid Tampering**
 - **Treating a common cause as a special cause**

Summary

- ❑ **Assessment of process stability**
- ❑ **Construction of averages control charts**
- ❑ **Differentiation between common and special causes**
- ❑ **Rational subgrouping**
- ❑ **Actions relevant to common and special causes**